

The Minimal Seesaw Model at the TeV Scale

He Zhang^{1,*} and Shun Zhou^{2,†}

¹*Department of Theoretical Physics, School of Engineering Sciences,
Royal Institute of Technology (KTH) – AlbaNova University Center,
Roslagstullsbacken 21, 106 91 Stockholm, Sweden*

²*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany*

We point out that the minimal seesaw model can provide a natural framework to accommodate tiny neutrino masses, while its experimental testability and notable predictiveness are still maintained. This possibility is based on the observation that two heavy right-handed Majorana neutrinos in the minimal seesaw model may naturally emerge as a pseudo-Dirac fermion. In a specific scenario, we show that the tri-bimaximal neutrino mixing can be produced, and only the inverted neutrino mass hierarchy is allowed. The low-energy phenomena, including non-unitarity effects in neutrino oscillations, neutrinoless double-beta decays and rare lepton-flavor-violating decays of charged leptons $\ell_\alpha \rightarrow \ell_\beta \gamma$, have been explored. The collider signatures of the heavy singlet neutrino are also briefly discussed.

PACS numbers: 11.30.Fs, 12.15.Ff, 14.60.St, 14.60.Pq

I. INTRODUCTION

Recently, a lot of attention has been focused on the experimental testability of seesaw models for neutrino masses at the CERN Large Hadron Collider (LHC). In the typical extension of the standard model (SM), three right-handed neutrinos are introduced and assigned large Majorana masses [1, 2, 3, 4]. In this case, the SM neutrinos acquire tiny Majorana masses via the type-I seesaw mechanism, i.e. the effective mass matrix of light neutrinos is given by $M_\nu \approx -M_D M_R^{-1} M_D^T$. Here the Dirac mass is naturally around the electroweak scale $M_D \sim \Lambda_{EW} \equiv 100$ GeV and the heavy Majorana neutrino masses are extremely large $M_R \sim 10^{10 \cdots 14}$ GeV. However, this type-I seesaw model suffers from the lack of testability, because right-handed neutrinos are too heavy to be produced in current collider experiments.

In order for the type-I seesaw model to be testable, we have to implement the structural cancellation condition $M_D M_R^{-1} M_D^T \approx \mathbf{0}$, which can lead to sub-eV neutrino masses but keep heavy Majorana neutrino masses as low as several hundred GeV [5, 6]. However, the fine-tunings of the structures of M_D and M_R are unavoidable, which seems unnatural. It has been pointed out that the heavy pseudo-Dirac neutrinos may account for tiny neutrino masses, such as in the so-called inverse seesaw models, which can be tested in collider experiments, and are free of the fine-tuning problem [7]. Nevertheless, more singlet fermions should be added in order to form pseudo-Dirac fermions together with singlet right-handed neutrinos. For instance, in the minimal version of inverse seesaw model, we must have four singlet fermions

to guarantee two massive light neutrinos [8]. Even more singlet fermions are required in the realization of multiple seesaw mechanisms [9], which are the direct generations of the type-I and the inverse seesaw models.

We propose that the minimal type-I seesaw model (MSM) with only two right-handed neutrinos may be the most natural candidate for realistic and testable neutrino mass models at the TeV scale. The reason is simply that these two right-handed neutrinos themselves can be combined together to make a Dirac fermion in the $U(1)$ symmetry limit. They may also be embedded into a two-dimensional representation of the discrete flavor symmetry groups, such as the permutation group S_4 [10, 11]. The soft symmetry-breaking terms then give rise to tiny neutrino masses, while the heavy pseudo-Dirac neutrino provides us with rich low-energy phenomena, e.g. the non-unitarity effects in neutrino oscillations and the lepton-flavor-violating decays of charged leptons. Furthermore, the tri-lepton signals $pp \rightarrow \ell_\alpha^\pm \ell_\beta^\pm \ell_\gamma^\mp \nu(\bar{\nu}) + \text{jets}$ of the pseudo-Dirac neutrino can be discovered at the LHC. Due to the minimal number of model parameters, the observables at low energies and in the collider experiments are intimately correlated with each other, and then serve as a cross test of our minimal TeV seesaw model.

The remaining part of this work is organized as follows. In Sec. II, we first present the structure of our model. The neutrino mass spectra and neutrino mixing patterns are discussed in detail in Sec. III. The implications for low-energy phenomena and possible collider signatures are addressed in Sec. IV. Finally, a brief summary is given in Sec. V.

II. THE MODEL

In the minimal seesaw model [12, 13], we extend the SM by introducing two heavy right-handed neutrinos,

*Electronic address: zhanghe@kth.se

†Electronic address: zhoush@mppmu.mpg.de

which are singlets under the SM gauge group. The Lagrangian relevant for neutrino masses is

$$-\mathcal{L}_{\text{mass}} = \bar{\nu}_L M_D n_R + \frac{1}{2} \bar{n}_R^c M_R n_R + \text{h.c.}, \quad (1)$$

where $\nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$ and $n_R = (n_{1R}, n_{2R})^T$ stand for left- and right-handed neutrinos, respectively. To effectively suppress the light neutrino masses while keeping heavy ones around the TeV scale, we additionally impose a global $U(1)$ symmetry on the generic Lagrangian, under which the charges of the SM lepton doublets are opposite to n_{2R} but equal to n_{1R} . With the help of such a lepton-number-like symmetry, the mass matrices M_D and M_R in Eq. (1) take very simple forms

$$M_D = v \begin{pmatrix} y_e & y_\mu & y_\tau \\ 0 & 0 & 0 \end{pmatrix}^T, \quad M_R = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} \quad (2)$$

with $v \simeq 174$ GeV being the vacuum expectation value of the Higgs field. In the flavor basis (ν_L, n_R^c) , the overall 5×5 neutrino mass matrix reads

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. \quad (3)$$

Note that the rank of \mathcal{M}_ν is two, so three light neutrinos are massless and two heavy Majorana neutrinos are degenerate in mass, as a consequence of the additional global symmetry. This can be easily verified by noting that the mass matrices in Eq. (2) satisfy the relation $M_D M_R^{-1} M_D^T = \mathbf{0}$, which implies that light neutrino masses are vanishing to all orders. Therefore, the masses of heavy right-handed neutrinos have nothing to do with light neutrinos and can be located at a relatively low scale, e.g., around the TeV scale. On the other hand, the symmetric matrix M_R can be diagonalized via an orthogonal transformation $V_R^T M_R V_R = \text{Diag}\{-M, M\}$ with

$$V_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \quad (4)$$

Thus the mass eigenstates of heavy right-handed neutrinos P_1 and P_2 possess identical masses but opposite CP parities, and they constitute a four-component Dirac particle $P = (P_1 + P_2)/\sqrt{2}$ with mass being M [14].

In order to accommodate light neutrino masses, one can add small perturbations to \mathcal{M}_ν , which softly break the global $U(1)$ symmetry. There are in principle four classes of soft-breaking perturbations to \mathcal{M}_ν , and the most general form of the perturbed neutrino mass matrix is given by

$$\mathcal{M}_\nu = \begin{pmatrix} \kappa_{ee} & \kappa_{e\mu} & \kappa_{e\tau} & v y_e & \varepsilon_e \\ \kappa_{\mu e} & \kappa_{\mu\mu} & \kappa_{\mu\tau} & v y_\mu & \varepsilon_\mu \\ \kappa_{\tau e} & \kappa_{\tau\mu} & \kappa_{\tau\tau} & v y_\tau & \varepsilon_\tau \\ v y_e & v y_\mu & v y_\tau & \mu' & M \\ \varepsilon_e & \varepsilon_\mu & \varepsilon_\tau & M & \mu \end{pmatrix}. \quad (5)$$

All the above perturbation terms $\kappa_{\alpha\beta}$, ε_α , μ' and μ (for $\alpha, \beta = e, \mu, \tau$) break the lepton number conservation, and

hence bring in neutrino masses proportional to the corresponding couplings. Some comments on the possible origins of these perturbations are in order:

(i) The κ term corresponds to a purely Majorana mass term of light neutrinos, which can be realized in a more complicated theory with additional contributions to neutrino masses. A typical example is the type-(I+II) seesaw model [15, 16, 17], where an $SU(2)$ triplet Higgs with mass much larger than the electroweak scale is involved. At lower-energy scales, the decoupling of the triplet Higgs will result in light neutrino masses together with non-standard interactions through the tree-level exchange of the neutral scalar [18]. Another possibility is to incorporate extra SM singlet or triplet fermions, which give birth to a neutrino mass operator similar to that in the type-I or type-III seesaw models [19]. Although feasible, the above mechanisms are always pestered with too many parameters, which render the models neither predictive nor economical.

(ii) The μ term in Eq. (5) is a bare Majorana mass insertion violating the lepton number by two units, which is also realized in the inverse seesaw framework [7, 20]. In the presence of the μ term, the light neutrino mass matrix can be obtained from the inverse seesaw formula

$$M_\nu \simeq \frac{\mu}{M^2} M_D M_D^T. \quad (6)$$

Hence the smallness of neutrino masses is attributed to both the small μ parameter and the ratio M_D/M . However, as pointed out in Ref. [8], at least two pairs of singlet heavy neutrinos are required in order to enhance the rank of \mathcal{M}_ν from two to four. One can also see this point from Eq. (6) that the rank of M_ν is exactly one, which definitely comes into conflict with the observed two mass-squared differences in neutrino oscillation experiments.

(iii) The μ' term in Eq. (5) does not contribute to neutrino masses at the tree level. However, it may radiatively generate neutrino masses via one-loop diagrams involving right-handed neutrinos and gauge bosons [5]. In addition, due to the corrections induced by the μ' term, the masses of P_1 and P_2 are not exactly equal, and thus the small mass splitting between heavy neutrinos could naturally make the resonant leptogenesis mechanism feasible [21]. In analogy with the μ -term corrections, the drawback is that only one light neutrino may acquire mass, and hence the μ' term is not phenomenologically adequate.

(iv) The ε term softly violates the extra $U(1)$ symmetry but enhances the rank of \mathcal{M}_ν from two to four, which is required by the neutrino oscillations. Such a coupling could be easily realized in grand unified theories, e.g. in the supersymmetric $SO(10)$ model with a very low $B-L$ scale [22]. As we will show later in this case, neutrino masses are naturally tiny, since they are proportional to ε , and are further suppressed by the mass ratio M_D/M . In the following, we will only concentrate on this particularly interesting pattern of neutrino mass generation, and figure out the phenomenological consequences in detail. The overall neutrino matrix can be obtained from Eq. (5) by setting all κ , μ and μ' to zero.

Without loss of generality, one can always redefine the lepton fields so as to remove the corresponding phases of y_α and M , leaving only ε_α complex. Therefore, we shall assume y_α and M to be real throughout the following discussions. The total neutrino matrix \mathcal{M}_ν can be diagonalized by the unitary transformation

$$V^\dagger \mathcal{M}_\nu V^* = \widehat{\mathcal{M}}_\nu \equiv \text{Diag}\{m_1, m_2, m_3, -M, M\}, \quad (7)$$

where V is a 5×5 unitary matrix, and m_i (for $i = 1, 2, 3$) are masses of three light neutrinos. In the leading-order approximation, the effective mass matrix of light neutrinos is given by the type-I seesaw formula

$$M_\nu \simeq -M_D M^{-1} M_D^T = -\varepsilon F^T - F \varepsilon^T, \quad (8)$$

where $\varepsilon = (\varepsilon_e, \varepsilon_\mu, \varepsilon_\tau)^T$ and $F = \omega(y_e, y_\mu, y_\tau)^T$ with $\omega \equiv v/M$. In general, M_ν can be diagonalized by a 3×3 unitary matrix as $U^\dagger M_\nu U^* = \widehat{M}_\nu \equiv \text{Diag}\{m_1, m_2, m_3\}$, and U is usually parametrized in the standard form

$$U = P_\phi R_{23}(\theta_{23}) P_\delta R_{13}(\theta_{13}) P_\delta^{-1} R_{12}(\theta_{12}) P_M, \quad (9)$$

where R_{ij} correspond to the elementary rotations in the $ij = 23, 13$, and 12 planes, θ_{ij} are the rotation angles, while $P_\delta = \text{Diag}\{1, 1, e^{i\delta}\}$ and $P_M = \text{Diag}\{1, e^{i\rho}, 1\}$ contain the Dirac and Majorana CP-violating phases, respectively. Note that only one Majorana phase is needed to parametrize U since one light neutrino is massless, which is the salient feature of MSM. The phases in $P_\phi = \text{Diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ are usually rotated away in the SM context but must be kept in the current model. It should be noticed that U is not exactly the matrix governing neutrino oscillations, even if we choose a basis where the charged-lepton mass matrix is diagonal. To clarify this point, we turn back to the 5×5 unitary matrix V in Eq. (7), which can be rewritten in a block form

$$V = \begin{pmatrix} N_{3 \times 3} & R_{3 \times 2} \\ S_{2 \times 3} & T_{2 \times 2} \end{pmatrix}. \quad (10)$$

The approximate expression of each block can be found in Ref. [23], and the relevant ones are

$$N \simeq \left(1 - \frac{1}{2} F F^\dagger\right) U, \quad R \simeq Q V_R, \quad (11)$$

with $Q \equiv (\mathbf{0}, F)$ being a 3×2 matrix. The flavor eigenstates of neutrinos are then the superpositions of light neutrino mass eigenstates $\nu_{mL} = (\nu_{1L}, \nu_{2L}, \nu_{3L})$ and the heavy one P . More specifically, the flavor eigenstates of light neutrinos can be expressed as $\nu_L \simeq N \nu_{mL} + F P^c$, which indicates that P^c mixes with left-handed neutrinos and enters into the weak interactions after the electroweak symmetry breaking. Therefore, the leptonic charged-current interactions in the mass basis read

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \bar{\ell}_L \gamma^\mu (N \nu_{mL} + F P^c) W_\mu^- + \text{h.c.} \quad (12)$$

It is obvious that the neutrino mixing matrix N appearing in the charged current is non-unitary. As we shall show, the mixing between light and heavy neutrinos will bring in several significant phenomenological consequences, in particular, when the scale of the heavy neutrino masses is accessible to future colliders.

III. NEUTRINO MASSES AND MIXING

First, we should consider the neutrino mass spectra and flavor mixing matrix. It is straightforward to obtain the light neutrino mass matrix from Eq. (8)

$$M_\nu = \omega \begin{pmatrix} 2\varepsilon_e y_e & \varepsilon_e y_\mu + \varepsilon_\mu y_e & \varepsilon_e y_\tau + \varepsilon_\tau y_e \\ \varepsilon_e y_\mu + \varepsilon_\mu y_e & 2\varepsilon_\mu y_\mu & \varepsilon_\mu y_\tau + \varepsilon_\tau y_\mu \\ \varepsilon_e y_\tau + \varepsilon_\tau y_e & \varepsilon_\mu y_\tau + \varepsilon_\tau y_\mu & 2\varepsilon_\tau y_\tau \end{pmatrix}, \quad (13)$$

where the irrelevant minus sign has been omitted. Since the experimental data on neutrino oscillations suggest the tri-bimaximal mixing pattern [24, 25], a μ - τ symmetry is particularly favorable in constructing the neutrino mass matrix. To this end, we assume here that the relations $\varepsilon_\mu = \varepsilon_\tau$ and $y_\mu = y_\tau$ hold. Defining $A = 2\omega\varepsilon_e y_e$, $B = \omega(\varepsilon_e y_\mu + \varepsilon_\mu y_e)$ and $C = 2\omega\varepsilon_\mu y_\mu$, one can rewrite Eq. (13) as follows

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & C \\ B & C & C \end{pmatrix}, \quad (14)$$

which can be further put into a 2×2 block form by a maximal rotation

$$M'_\nu = R_{23}\left(\frac{\pi}{4}\right) M_\nu R_{23}^T\left(\frac{\pi}{4}\right) = \begin{pmatrix} A & \sqrt{2}B & 0 \\ \sqrt{2}B & 2C & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (15)$$

We have found that only the inverted mass hierarchy $m_2 > m_1 > m_3 = 0$ is compatible with a maximal mixing pattern in the $2 \leftrightarrow 3$ sector. The left-up block matrix in Eq. (15), denoted as \tilde{M}'_ν , is a general 2×2 symmetric matrix, which can be diagonalized as $U_0^\dagger \tilde{M}'_\nu U_0^* = \text{Diag}\{m_1, m_2\}$ with

$$U_0 = \begin{pmatrix} c_\theta & s_\theta e^{-i\phi} \\ -s_\theta e^{i\phi} & c_\theta \end{pmatrix} \begin{pmatrix} e^{i\psi_1} & 0 \\ 0 & e^{i\psi_2} \end{pmatrix}. \quad (16)$$

Here $c_\theta \equiv \cos \theta$ and $s_\theta \equiv \sin \theta$ have been defined. After a lengthy but straightforward calculation, one can obtain

$$\tan \phi = \frac{X}{Y}, \quad \tan 2\theta = \frac{\sqrt{2(X^2 + Y^2)}}{|A|^2 - 4|C|^2}, \quad (17)$$

where

$$\begin{aligned} X &= \text{Im}(B)\text{Re}(A - 2C) - \text{Re}(B)\text{Im}(A - 2C), \\ Y &= \text{Im}(B)\text{Im}(A + 2C) + \text{Re}(B)\text{Re}(A + 2C), \end{aligned} \quad (18)$$

together with the mixing angles $\theta_{13} = 0$ and $\theta_{23} = 45^\circ$ defined in Eq. (9). The non-vanishing neutrino mass eigenvalues are given by

$$\begin{aligned} m_1 &= \left| Ac_\theta^2 - \sqrt{2}Bs_\theta^2 e^{-i\phi} + 2Cs_\theta^2 e^{-2i\phi} \right|, \\ m_2 &= \left| As_\theta^2 + \sqrt{2}Bs_\theta^2 e^{-i\phi} + 2Cc_\theta^2 e^{-2i\phi} \right|. \end{aligned} \quad (19)$$

In addition, the phase difference $\Delta\psi \equiv \psi_2 - \psi_1$ reads

$$\Delta\psi = \frac{1}{2} \arg \left(\frac{As_\theta^2 + \sqrt{2}Bs_\theta^2 e^{-i\phi} + 2Cc_\theta^2 e^{-2i\phi}}{Ac_\theta^2 - \sqrt{2}Bs_\theta^2 e^{-i\phi} + 2Cs_\theta^2 e^{-2i\phi}} \right). \quad (20)$$

In comparison with the standard parametrization in Eq. (9), we can observe that $\rho = \Delta\psi - \phi$ is just the physical Majorana phase. Once the identity $\sqrt{X^2 + Y^2} = 2(|A|^2 - 4|C|^2)$ is fulfilled, the tri-bimaximal mixing pattern with $\theta_{12} = \theta \approx 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0$ is reproduced in the leading order. The vanishing θ_{13} can be regarded as a consequence of the exact $\mu \leftrightarrow \tau$ symmetry. If we relax this assumption, i.e. $\varepsilon_\mu \neq \varepsilon_\tau$ and (or) $y_\mu \neq y_\tau$, both non-vanishing mixing angle θ_{13} and CP-violating phase δ can be accommodated.

As we discussed above, one peculiar feature in our model is that the mixing matrix of light neutrinos N is no longer unitary. Adopting the parametrization of non-unitary leptonic mixing matrix $N = (1 - \eta)U$ in Ref. [26], we can see from Eq. (11) that

$$\eta = \frac{1}{2} F F^\dagger = \frac{\omega^2}{2} \begin{pmatrix} y_e^2 & y_e y_\mu & y_e y_\tau \\ y_e y_\mu & y_\mu^2 & y_\mu y_\tau \\ y_e y_\tau & y_\mu y_\tau & y_\tau^2 \end{pmatrix}. \quad (21)$$

The $\mu \leftrightarrow \tau$ symmetric feature of the effective neutrino mass matrix M_ν indicates $\eta_{e\mu} \sim \eta_{e\tau}$. Since the most stringent experimental constraints come from the lepton-flavor-violating (LFV) decay $\mu \rightarrow e\gamma$, severe bounds on $\eta_{e\mu}$ and $\eta_{e\tau}$ are expected.

It is worthwhile to note that the ratios $\varepsilon_\mu/\varepsilon_e$ and y_μ/y_e are of great importance in our model. For a given mass M (or equivalently $\omega \equiv v/M$) and also the mass scale of light neutrinos $\omega y_e |\varepsilon_e|$, all the other observables are determined by these ratios. One can always appropriately rescale ε_α and y_α to get the desired masses m_i , keeping the ratios $\varepsilon_\mu/\varepsilon_e$ and y_μ/y_e unchanged. Therefore, we shall mainly concentrate on the relative ratios among model parameters in the following discussions.

In FIG. 1, we show the allowed parameter space of the model within 3σ C.L. In our numerical analysis, we take $M = 1$ TeV for example, as well as the values of neutrino masses and mixing angles from Ref. [27]. We also include the experimental constraints on the non-unitary effects coming from universality tests of weak interactions, rare leptonic decays, invisible width of the Z -boson and neutrino oscillation data [28]. Technologically, we randomly choose the values of $(\varepsilon_e, \varepsilon_\mu, y_e, y_\mu)$ and their corresponding phases, while the data sample reproducing all the neutrino masses and mixing angles within 3σ confidence

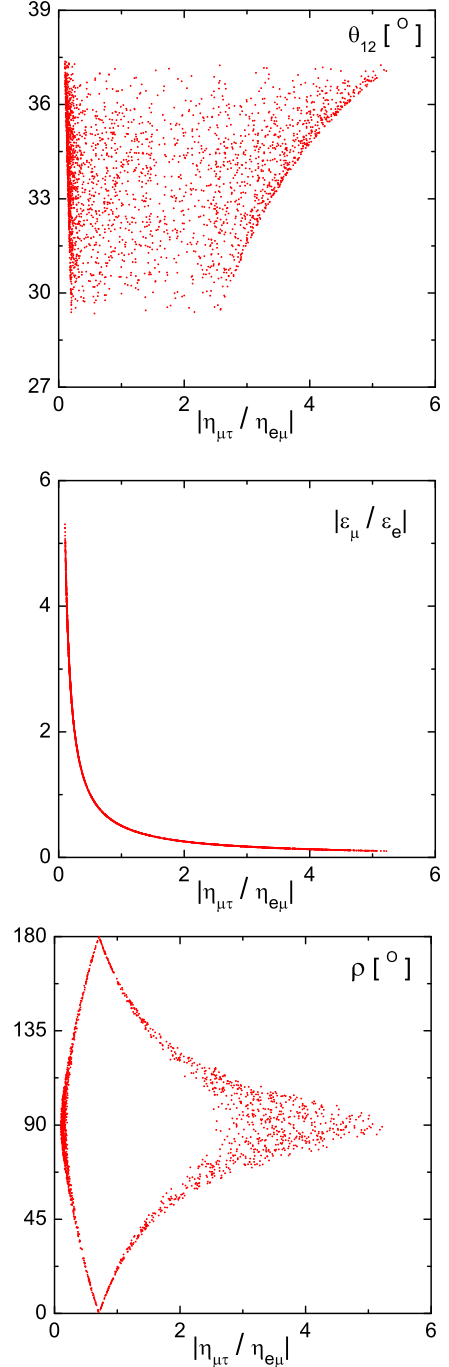


FIG. 1: The parameter space of the neutrino mixing angle θ_{12} , the Majorana phase ρ , and the ratios $(\varepsilon_\mu/\varepsilon_e, \eta_{\mu\tau}/\eta_{e\mu})$.

ranges will be kept. The absolute scales of ε_α and y_α (for $\alpha = e, \mu$) are also checked to be consistent with the experimental bounds on η . Some comments are in order:

- From the uppermost plot, one can see that the ratio $|\eta_{\mu\tau}/\eta_{e\mu}| = |y_\mu/y_e|$ is strictly constrained by θ_{12} at large values, and the maximal value of the ratio is close to 5. In this case, the current bound $|\eta_{e\mu}| < 6 \times 10^{-5}$ suggests a rather stringent bound $|\eta_{\mu\tau}| \lesssim$

3×10^{-4} . However, as pointed out in Ref. [28], in the case with $M < \Lambda_{\text{EW}}$ but above a few GeV, the severe constraints from Z and W decays do not apply since the unitarity is restored. In this special case the upper bound $|\eta_{e\mu}| < 9 \times 10^{-4}$ indicates that there is no general constraint on $|\eta_{\mu\tau}|$.

- The correlation between $|\varepsilon_\mu/\varepsilon_e|$ and $|\eta_{\mu\tau}/\eta_{e\mu}|$ is illustrated in the middle plot. The approximate relation $|\varepsilon_\mu/\varepsilon_e| \times |\eta_{e\mu}/\eta_{\mu\tau}| \approx 0.5$ reflects the fact that $|A|$ and $2|C|$ in Eq. (15) are comparable in magnitude to ensure the experimental result $m_1 \simeq m_2$. To generate a large but non-maximal mixing angle θ_{12} , the relation $|B| \ll |A|$ must hold, which is only possible if the relative phase between A and C (or equivalently ε_e and ε_μ) is around 180° . However, the exact identity $|A| = 2|C|$ is forbidden due to the mixing angle θ in Eq. (17), which exhibits the strict experimental constraints on model parameters.
- In the last plot, we show the allowed ranges of the Majorana phase ρ versus the ratio $|\eta_{\mu\tau}/\eta_{e\mu}|$. As we have discussed, the requirement $A \simeq -2C$ also sets a strong constraint on the relative phase between ε_e and ε_μ . For a sizable $\eta_{\mu\tau}$, which is phenomenologically interesting, $\rho \sim 90^\circ$ can be read from the plot. For intermediate values of $\eta_{\mu\tau}$, we can see that ρ deviates significantly from 90° . In principle, the whole range $[0, 180^\circ]$ of ρ is allowed.

As a consequence of the minimal number of model parameters, the mixing angle θ_{12} , the Majorana phase ρ and the non-unitarity parameters $|\eta_{\alpha\beta}|$ are now related to one another. Furthermore, the introduction of non-vanishing θ_{13} and CP-violating phase is straightforward by relaxing the μ - τ symmetry.

IV. FURTHER DISCUSSIONS

Now we shall address the possibly intriguing low-energy phenomena of our model, as well as the signals of the heavy neutrinos at the LHC.

a. Rare LFV decays mediated by the heavy neutrino. The heavy neutrinos entering into the charged-current interaction will contribute to the LFV decays of charged leptons. For instance, the decay channel $\ell_\alpha \rightarrow \ell_\beta \gamma$ can be mediated by heavy neutrinos [29]. In the standard type-I seesaw scenario or MSM, one has approximately $(M_D/M)^2 = \mathcal{O}(mM^{-1})$, and therefore $\text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma) \propto \mathcal{O}(m^2)$ indicates a strong suppression of LFV decay rates, where m and M denote respectively the scales of light and heavy neutrino masses. However, in our model, one can have sizeable M_D/M without facing the difficulty of neutrino mass generation since they are suppressed by the perturbations ε_α . Thus appreciable LFV rates could be obtained even for strictly massless light neutrinos [30].

b. Neutrinoless double beta decays ($0\nu 2\beta$). The light Majorana neutrinos contribute to $0\nu 2\beta$ decays,

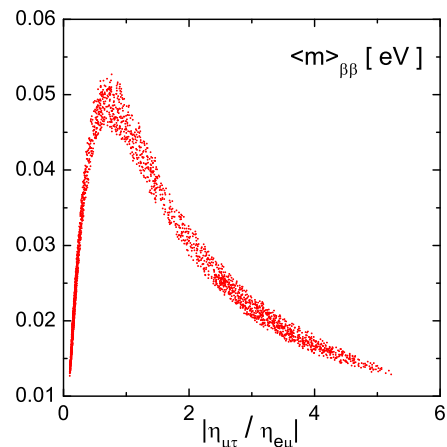


FIG. 2: The allowed region of the effective neutrino mass $\langle m \rangle_{\beta\beta}$ versus the ratio $|\eta_{\mu\tau}/\eta_{e\mu}|$.

which serve as the unique tool to discriminate between Majorana and Dirac nature of massive neutrinos. The relevant quantity is the effective neutrino mass $\langle m \rangle_{\beta\beta} \equiv |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|$, which in our model with $m_3 = 0$ can be evaluated as $\langle m \rangle_{\beta\beta} = |m_1 U_{e1}^2 + m_2 U_{e2}^2|$. Because of $m_2 > m_1 = \sqrt{|\Delta m_{31}^2|} \approx 0.05$ eV, the two terms in the $\langle m \rangle_{\beta\beta}$ are comparable in magnitude. In this case, the Majorana phase plays a key role in determining $\langle m \rangle_{\beta\beta}$. For example, if ρ is far away from 90° , the contributions from these two terms should be added constructively, and one can then expect a large value $\langle m \rangle_{\beta\beta} \sim 0.05$ eV. In FIG. 2 we have shown the allowed region of $\langle m \rangle_{\beta\beta}$. The result is in agreement with that in FIG. 1. It is worth noting that heavy Majorana neutrinos are nearly degenerate in mass, i.e. they form a pseudo-Dirac neutrino, so their contributions to $\langle m \rangle_{\beta\beta}$ can be neglected. Interestingly, the next-generation $0\nu 2\beta$ decay experiments are expected to probe $\langle m \rangle_{\beta\beta}$ with the accuracy of $10 - 50$ meV, so our model can be tested experimentally in the near future.

c. Search for the tri-lepton signals at the LHC. As shown in Eq. (12), the heavy singlet P couples to the gauge sector of the SM, and thus if kinematically accessible, could be produced at hadron colliders. In the case $M > M_H$ (where M_H denotes the Higgs mass), the heavy neutrino can decay in the channels $P \rightarrow \ell^+ + W^-$, $P \rightarrow \bar{\nu} + Z$, and $P \rightarrow \bar{\nu} + H$. Now that the heavy neutrinos form a pseudo-Dirac particle, we shall focus our attention on the lepton-number-conserving processes induced by it. For example, one very interesting and prospective channel is the production of three charged leptons with missing energy [31], i.e. $pp \rightarrow \ell_\alpha^\pm \ell_\beta^\pm \ell_\gamma^\mp \nu(\bar{\nu}) + \text{jets}$. Another possible one is the pair production of charged leptons in different flavors and without missing energy, i.e. $pp \rightarrow \ell_\alpha^\pm \ell_\beta^\mp + \text{jets}$. However, it is difficult to make significant observation in this channel at the LHC due to the large SM background [32].

V. SUMMARY

In this work, we have considered a novel minimal seesaw model, which is quite natural to bring down the typical seesaw mechanism to the electroweak scale. Our model is minimal among the fermionic seesaw scenarios in the sense of particle content, since only two right-handed neutrinos are introduced. However, quite different from the traditional MSM, the heavy right-handed neutrinos in the current consideration is located around the TeV scale, and possess sizable Yukawa couplings, which make them observable, in particular at the LHC. Furthermore, light neutrino masses are protected by a global $U(1)$ symmetry, and thus free from radiative instability. Since the model contains only two heavy singlet neutrinos, it is very predictive compared to the other seesaw mechanisms. We have shown that, in the assumption of a $\mu \leftrightarrow \tau$ symmetry,

the tri-bimaximal pattern of lepton flavor mixing can be reproduced and only the inverted neutrino mass hierarchy is allowed. The light neutrino masses, the Majorana phase and non-unitarity parameters are connected in our model, and their non-trivial correlations could be tested through a combined analysis of the future long-baseline neutrino oscillation experiments, the rare LFV decays, the $0\nu 2\beta$ decays and the collider signals at the LHC.

Acknowledgments

One of the authors (S.Z.) is indebted to G.G. Raffelt for warm hospitality at Max-Planck-Institut für Physik. This work was supported by the Royal Institute of Technology (KTH), project no. SII-56510 (H.Z.), and the Alexander von Humboldt Foundation (S.Z.).

-
- [1] P. Minkowski, Phys. Lett. **B67**, 421 (1977).
 - [2] T. Yanagida, in *Proc. Workshop on the Baryon Number of the Universe and Unified Theories*, edited by O. Sawada and A. Sugamoto (1979), p. 95.
 - [3] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (1979), p. 315.
 - [4] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
 - [5] A. Pilaftsis, Z. Phys. **C55**, 275 (1992), hep-ph/9901206.
 - [6] J. Kersten and A. Y. Smirnov, Phys. Rev. **D76**, 073005 (2007), arXiv:0705.3221.
 - [7] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. **D34**, 1642 (1986).
 - [8] M. Malinský, T. Ohlsson, Z.-z. Xing, and H. Zhang, Phys. Lett. **B679**, 242 (2009), arXiv:0905.2889.
 - [9] Z.-z. Xing and S. Zhou, Phys. Lett. **B679**, 249 (2009), arXiv:0906.1757.
 - [10] Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. **D25**, 1895 (1982).
 - [11] H. Zhang, Phys. Lett. **B655**, 132 (2007), hep-ph/0612214.
 - [12] P. H. Frampton, S. L. Glashow, and T. Yanagida, Phys. Lett. **B548**, 119 (2002), hep-ph/0208157.
 - [13] W.-l. Guo, Z.-z. Xing, and S. Zhou, Int. J. Mod. Phys. **E16**, 1 (2007), hep-ph/0612033.
 - [14] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
 - [15] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).
 - [16] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).
 - [17] R. N. Mohapatra and G. Senjanović, Phys. Rev. **D23**, 165 (1981).
 - [18] M. Malinský, T. Ohlsson, and H. Zhang, Phys. Rev. **D79**, 011301(R) (2009), arXiv:0811.3346.
 - [19] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. **C44**, 441 (1989).
 - [20] M. Malinský, T. Ohlsson, and H. Zhang (2009), arXiv:0903.1961.
 - [21] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. **D72**, 113001 (2005), hep-ph/0506107.
 - [22] M. Malinský, J. C. Romao, and J. W. F. Valle, Phys. Rev. Lett. **95**, 161801 (2005), hep-ph/0506296.
 - [23] J. Schechter and J. W. F. Valle, Phys. Rev. **D25**, 774 (1982).
 - [24] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. **B530**, 167 (2002), hep-ph/0202074.
 - [25] Z.-z. Xing, Phys. Lett. **B533**, 85 (2002), hep-ph/0204049.
 - [26] G. Altarelli and D. Meloni, Nucl. Phys. **B809**, 158 (2009), arXiv:0809.1041.
 - [27] T. Schwetz, M. A. Tórtola, and J. W. F. Valle, New J. Phys. **10**, 113011 (2008), arXiv:0808.2016.
 - [28] S. Antusch, J. P. Baumann, and E. Fernández-Martínez, Nucl. Phys. **B810**, 369 (2009), arXiv:0807.1003.
 - [29] A. Ilakovac and A. Pilaftsis, Nucl. Phys. **B437**, 491 (1995), hep-ph/9403398.
 - [30] J. Bernabéu, A. Santamaria, J. Vidal, A. Mendez, and J. W. F. Valle, Phys. Lett. **B187**, 303 (1987).
 - [31] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. **B672**, 158 (2009), arXiv:0809.2096.
 - [32] F. del Aguila, J. A. Aguilar-Saavedra, and R. Pittau, JHEP **10**, 047 (2007), hep-ph/0703261.